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Persistent current and low-field magnetic susceptibility in n-fold twisted Moebius strips

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Abstract : We study persistent currents and sign of these currents in the limit $\phi \to 0$ for n-fold twisted Moebius strips threaded by a slowly varying magnetic flux ϕ concerning the dependence of the total number of electrons N_e, chemical potential μ, number of twist n and randomness. In odd-fold twisted Moebius strips current gets $\phi_0/2$ flux-quantum periodicity only when $v_\perp = 0$ (transverse hopping stregth), while, it shows ϕ_0 flux-quantum periodicity as long as the electrons are allowed to hop along the transverse direction ($v_\perp \neq 0$). On the other hand, current always exhibits ϕ_0 flux-quantum periodicity for even-fold twisted Moebius strips, irrespective of the transverse hopping strength ($v_\perp$). The sign of the low-field currents also strongly depends on the number of twist n. For $v_\perp = 0$ the sign of the low-field currents can be mentioned exactly in old-fold twisted Moebius strips those are characterized by fixed N_e only. In absence of any impurity current shows always the diamagnetic nature irrespective of N_e *i.e.*, whether N_e is odd or even, while, in presence of impurity it exhibits respectively the diamagnetic and the paramagnetic behavior for the systems with odd and even N_e. The sign of these low-field currents cannot be predicted exactly when the electrons are allowed to hop along the transverse direction. Since then it strongly depends on the N_e, μ and specific realization of randomness.

Keywords : Persistent current, Magnetic Susceptibility and Moebius Strip

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1. Introduction

With the advance in nanoscience and technology, the fabrication of sub-micrometer devices has become possible and an emerging tendency in modern science is to propose and to investigate systems containing smaller and smaller structures. The resulting systems approach the so-called mesoscopic regime where the quantum phase coherence leads to important corrections to the electronic properties of the devices. The most exciting in the view of applications are the electronic properties of such systems. In what follows, we shall concentrate to one of the most striking evidences of quantum phase coherence, the so-called persistent current in nomal conducting loops. The pioneering work of Büttiker, Imry and Landauer [1] has predicted that a small conducting isolated ring threaded by a slowly varying magnetic flux ϕ carries an equilibrium current which *persists* in the system and varies periodically with the applied flux. Since then the phenomenon of persistent current in mesoscopic systems has been studied quite extensively in the literature both theoretically [2-8] and experimetally [9-13] At sufficiently low temperatures, the electron transport is completely phase coherent and in these small systems the energy levels are discrete The phase coherence of the electrons and the discreteness of the energy levels play important role in the existence of the persistent current in these systems We assume that the magnetic field doesn't penetrate the ring circumference anywhere and thus the electrons always move in field-free space This system can be mapped onto the band structure problem of a one-dimensional infinite crystal The idea is that once the electron circles the ring, it gets exactly the same potential as before, but now has acquired an additional phase $2\pi\phi/\phi_0$ Thus a close connection is obtained between the states of an electron in a loop and the 1D Bloch problem by identifying $2\pi\phi/\phi_0$ with kL [2], where L is the circumference of the loop In a recent experiment, Tanda *et al* [14] have fabricated a microscopic $NbSe_3$ Moebius strip and it raises some interesting questions regarding the topological effect on the persistent currents and the low-field magnetic susceptibility in twisted Moebius geometries Here we investigate in detail all the above mentioned properties for n-fold twisted Moebius strips

This paper is specifically organized as follows. In section 2, we describe in detail the characteristic properties of the persistent currents in n-fold twisted Moebius strips specified by the fixed chemical potential μ , instead of the fixed number of electrons N_e The topological effect on the sign of the low-field currents are investigated in section 3 Finally we conclude our results in section 4.

2. Persistent current in n-fold twisted Moebius strips with fixed μ

In this section we focus our study on the behavior of persistent currents in n-fold twisted Moebius strips concerning the value of n and try to emphasize some intuitive points relevant to the flux periodicities of these currents. Figure 1 shows the schematic view of a simplest one-fold (n = 1) twisted Moebius strip which encloses a magnetic flux ϕ We assume that the magnetic field does not penetrate the circumference of the strip anywhere and hence the additional Zeeman term should not be taken into account during the

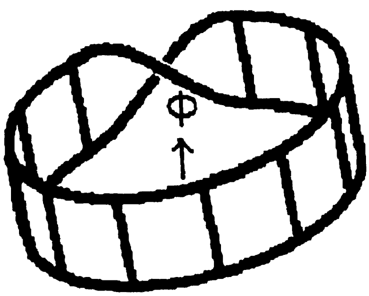

Figure 1. Schematic view of a one-fold (n = 1) twisted Moebius strip threaded by a magnetic flux

calculations. To describe such a Moebius strip with N rungs we use the tight-binding model Hamiltonian which can be written within the non-interacting picture in this form

$$H = \sum_{i=1}^{2N} \varepsilon_i c_i^\dagger c_i + v \sum_{i=1}^{2N} \left(e^{i\theta} c_i^\dagger c_{i+1} + h.c \right) + v_\perp \sum_{i=1}^{2N} c_i^\dagger c_{i+N} \tag{1}$$

where ε_i's are the on-site energies, $c_i^\dagger (c_i)$ is the creation (annihilation) operator of the electron at site i, the phase factor $\theta = 2\pi\phi/N$, v is the nearest-neighbor hopping strength along the longitudinal direction and $v_\perp$ is the perpendicular hopping strength To introduce the impurities in such a strip we set the site energies (ε_i's) in the form of incommensurate potentials through the relation $\varepsilon_i = \sum_i W \cos(i\lambda\pi)$, where λ is an irrational number, and as a typical example we take it as the golden mean $\left(1+\sqrt{5}\right)/2$ and W is the strength of the disorder Setting $\lambda = 0$, we get back the pure system with identical site potential W In the obvious reason we do not take any disorder averaging since the impurities are given from this incommensurate distribution function Here we use the convention $c_{i+2N} = c_i$ and for the sake of simplicity we take the units $c = e = h = 1$ All the calculations in this article are performed only at absolute zero temperature ($T = 0$).

Now we concentrate our study on the characteristics of the persistent currents in n-fold twisted Moebius strips those are described with the fixed chemical potential μ, instead of specifying the total number of electrons N_e The current carried by an energy eigenstate is obtained by taking the first order derivative of the energy with respect to the

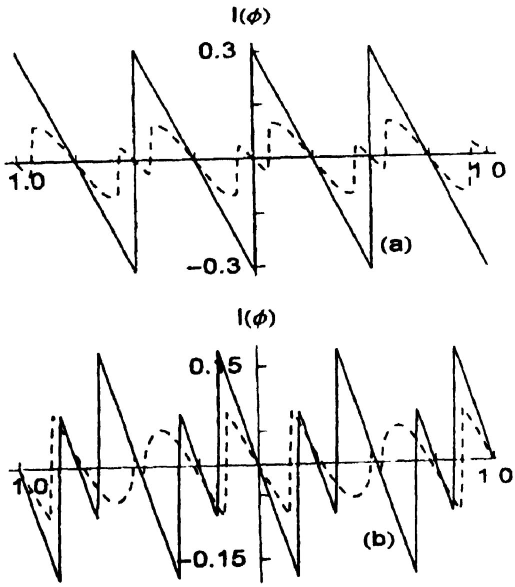

Figure 2. *$I(\Phi)$ versus ϕ* curves both for the perfect ($W = 0$) and the disordered ($W = 1$) three-fold ($n = 3$) twisted Moebius strips characterized by fixed chemical potential $\mu = 0$ with (a) $v_\perp = 0$ and (b) $v_\perp = -1$ The solid and the dotted curves correspond to the results for the perfect and the disordered strips, respectively

flux ϕ [15] The total current of the system is then determined by adding the individual contributions of all the energy levels having energies less than or equal to μ As the chemical potential is fixed, the total number of electrons N_e varies as a function of ϕ, in contrary to the systems those are described with fixed N_e where the chemical potential changes with the flux ϕ We will describe all the essential features of our study for the two limiting cases of the transverse hopping strength ($v_\perp$) One is the case where $v_\perp$ = 0 and for the other case we take $v_\perp \neq 0$.

Let us first describe the behaviour of the persistent currents in odd-fold twisted Moebius strips for the two limiting cases As illustrative examples, in Figure 2 we display the persistent currents for some 3-fold (n = 3) twisted Moebius strips considering the system size N = 40 and the chemical potential μ = 0 Figure 2(a) corresponds to the currents for the zero transverse hopping strengtn, while, Figure 2(b) corresponds to the currents for the non-zero transverse hopping with strength $v_\perp = -1$ The solid and the dotted curves are respectively for the perfect (W = 0) and the dirty (W = 1) strips In the impurity-free strips, sharp transitions appear in the persistent currents (solid curves of Figure 2(a)) for some particular field points associated with the degeneracy of the energy eigenstates at these respective field points On the other hand, in the presence of impurity all the degeneracies move out and the currents vary continuously (dotted curves of Figure 2(a)) with the flux ϕ and get much reduced values The removal of such degeneracy can be very well understood from the standard interpretation of the perturbation theory and the reduction of the current amplitude is due to the localization effect of the energy eigenstates in presence of the impurity More kinks appear (see the solid curve of Figure 2(b)) when

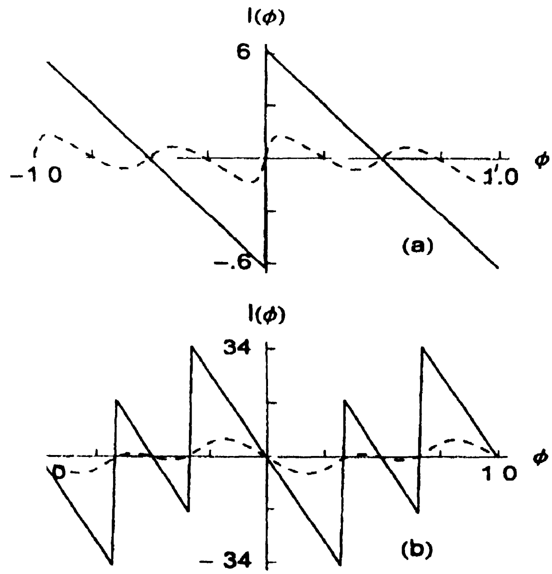

Figure 3. $I(\phi)$ *versus* ϕ curves both for the perfect (W = 0) and the disordered (W = 1) two-fold (n = 2) twisted Moebius strips characterized by fixed chemical potential $\mu = 0$ with (a) $v_\perp = 0$ and (b) $v_\perp = -1$ The solid and the dotted curves correspond to the results for the perfect and the disordered strips, respectively

the electrons are allowed to hop along the transverse direction in the perfect strip The appearance of these additional kinks at different field points depends on the choices of the chemical potential μ . All these kinks disappear as long as the impurities are introduced in the system. The most significant observation is that the current exhibits $\phi_0/2$ flux-quantum periodicity only when $v_\perp = 0$, but, for non-zero value of it the current shows the similar ϕ_0 periodic nature as in a regular multi-channel cylinder [15] This can be understood as follows. In odd-fold twisted Moebius strips if $v_\perp = 0$ then an electron which moves along the longitudinal direction traverses twice the path length to reach its initial position and encloses 2ϕ flux Accordingly, the current gets $\phi_0/2$ flux-quantum periodicity On the other hand, for $v_\perp \neq 0$ the electron can hop along the transverse direction and hence we get the usual ϕ_0 periodic currents

When the number of twist n becomes an even then a Moebius strip becomes a regular two-channel mesoscopic cylinder. For such a case we get the quite similar behavior of persistent currents as obtained in small regular cylinders those have been studied previously in the literature [15] As representative examples, in Figure 3 we plot the characteristic features of persistent currents for some 2-fold twisted Moebius strips for both the two limiting cases considering the same system size and the chemical potential as described earlier The solid and the dotted curves correspond to the same meaning as in Figure 2 As expected both for the two limiting cases we get only ϕ_0 periodic currents Thus we can predict that the number of twist and the transverse hopping have significant role in the determination of the periodicity of the persistent currents Quite similar features are also visible for the strips described with fixed number of electrons N_e [15] which we do not describe here

3. Characterization of the sign of the low-field currents

The determination of the sign of the low-field currents is an another important issue for this particular study. This can be done by calculating the magnetic susceptibility $\chi(\phi)$ in the limit $\phi \to 0$ The general expression of the magnetic susceptibility is expressed in the form

$$\chi(\phi) = \frac{N^3}{16\pi^2} \frac{\partial I(\phi)}{\partial \phi} \tag{2}$$

Here we describe quite briefly about the sign of the low-field currents both for the strips those are characterized with fixed N_e and μ , concerning the values of n and $v_\perp$.

Let us first consider the case where the strips are characterized by the fixed number of electrons N_e. For the odd-fold twisted Moebius strips and the zero transverse hopping strength the sign of the low-field currents is exactly predictable It is always diamagnetic in nature for impurity-free Moebium strips irrespective of N_e *i e.,* whether N_e is even or odd On the other hand, in the presence of impurity the sign of the low-field currents becomes diamagnetic and paramagnetic for the strips containing odd and even N_e respectively. This is the general feature of any regular one-channel, since here $v_\perp = 0$ mesoscopic loop described by the fixed N_e and is independent of the specific realization of the disordered configurations [15]. When $v_\perp \neq 0$, the systems are not treated as one-channel loops and the sign of the currents cannot be specified exactly even in the absence of any impurity. Then it depends on N_e and for disordered systems it also depends significantly on the specific realization of the disordered configurations. For the case of even-fold twisted Moebius strip as the system maps into the regular multi-channel cylinder the sign of the low-field currents is no longer predictable exactly, even if the system is a perfect one.

Similar arguments are also valid for the Moebius strips described with fixed chemical potential μ, instead of N_e, but in this case the sign of the low-field currents cannot be predicted even for the Moebius strips with odd n and $v_\perp = 0$ *i.e.*, when the systems become one-channel systems. Both for the Moebius strips with odd and even n, the sign of the low-field currents strongly depends on the choices of μ and in dirty systems it depends on the specific realization of the disordered configurations.

4. Conclusion

In conclusion, we have studied the characteristic features of the persistent currents and the sign of these currents in the low-field limit for some n-fold twisted Moebius strips concerning the dependence of the total number of electrons N_e, chemical potential μ number of twist n and randomness. We have used the tight-binding formulation to compute the results and all the calculations have been performed at absolute zero temperature (T = 0). Several kink-like structures appear in the persistent currents at different field points for the impurity-free Moebius strips depending on the choices of μ. These kinks arise due to the existence of the degenerate energy eigenstates at these respective field points and they disappear as long as the impurities are switched on and accordingly, the current varies continuously with the flux ϕ. The most significant observation is the existence of the $\phi_0/2$ flux-quantum periodicty in odd-fold twisted Moebius strips for the limiting case when $v_\perp = 0$. In such a situation an electron which moves along the longitudinal direction encircles the loop twice to reach its original position and therefore it encloses 2ϕ flux which reveals $\phi_0/2$ periodic currents. On the other hand, in all other cases we get the usual ϕ_0 periodic currents.

At the end, we have concentrated on the sign of the low-field currents. For the Moebius strips with odd n and $v_\perp = 0$, the systems can be treated as one-channel systems and only for such cases the sign of the low-field currents can be predicted exactly provided the systems are described by fixed number of electrons N_e. In impurity-free systems current shows only the diamagnetic sign whether the systems contain odd or even N_e But in the dirty systems the low-field current shows the diamagnetic and the paramagnetic nature for the systems with odd and even N_e, respectively. For all other cases the systems are equivalent to multi-channel systems and the sign of the currents cannot be predicted exactly Then it strongly depends on N_e, μ and the specific realization of the disordered configurations.

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